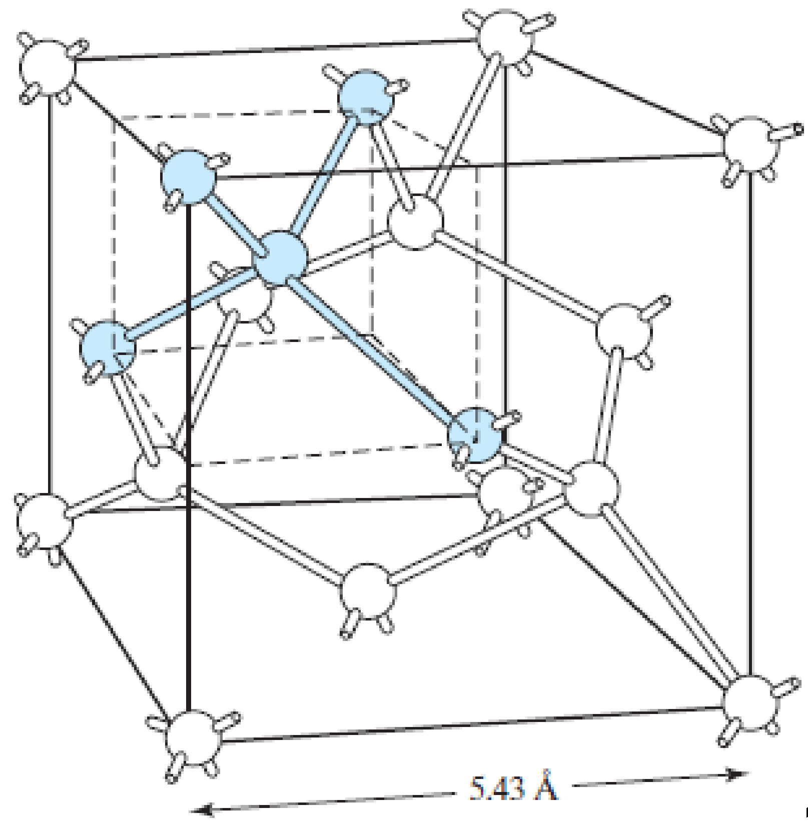
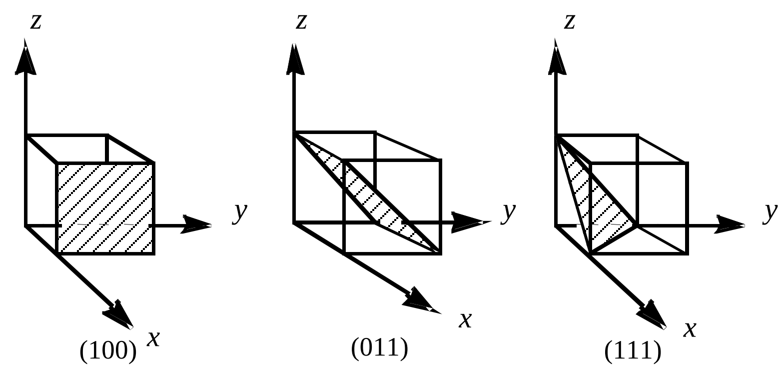
Chapter 1 Electrons and Holes in Semiconductors

1.1 Silicon Crystal Structure

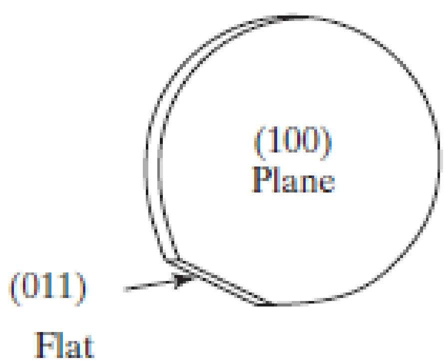
- ***Unit cell*** of silicon crystal is cubic.
- ***Each Si atom has 4 nearest neighbors.***

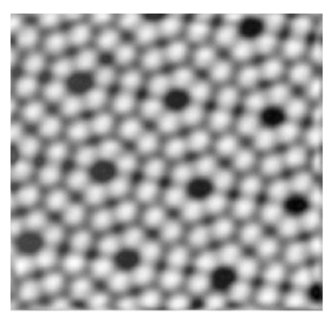


Silicon Wafers and Crystal Planes



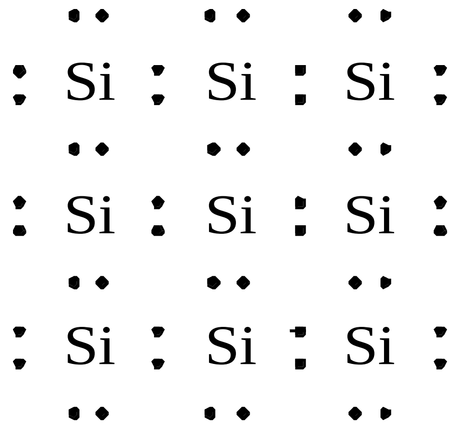
- The standard notation for crystal planes is based on the cubic unit cell.



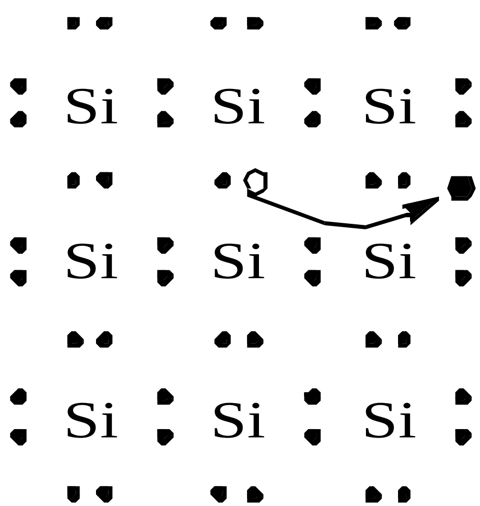

Si (111) plane

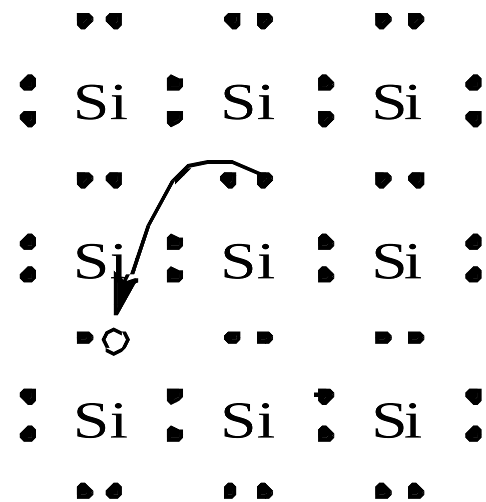
- Silicon wafers are usually cut along the (100) plane with a flat or notch to help orient the wafer during IC fabrication.

1.2 Bond Model of Electrons and Holes



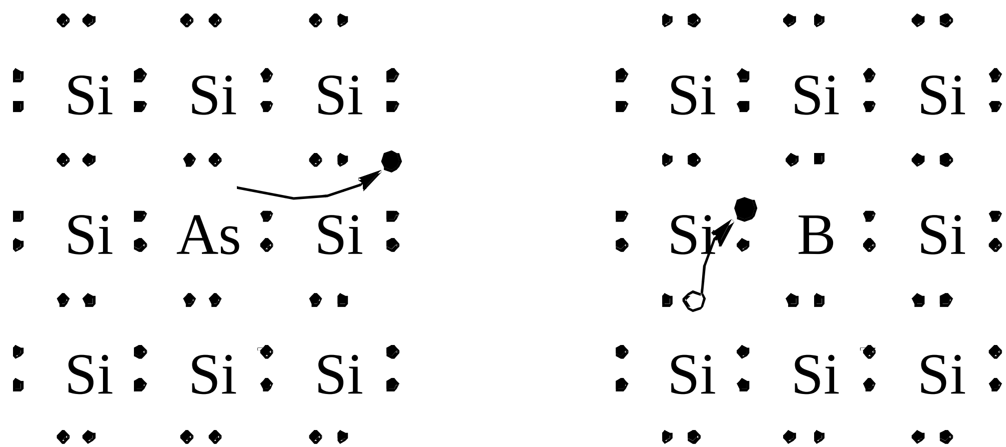
- Silicon crystal in a two-dimensional representation.





- When an electron breaks loose and becomes a ***conduction electron***, a ***hole*** is also created.

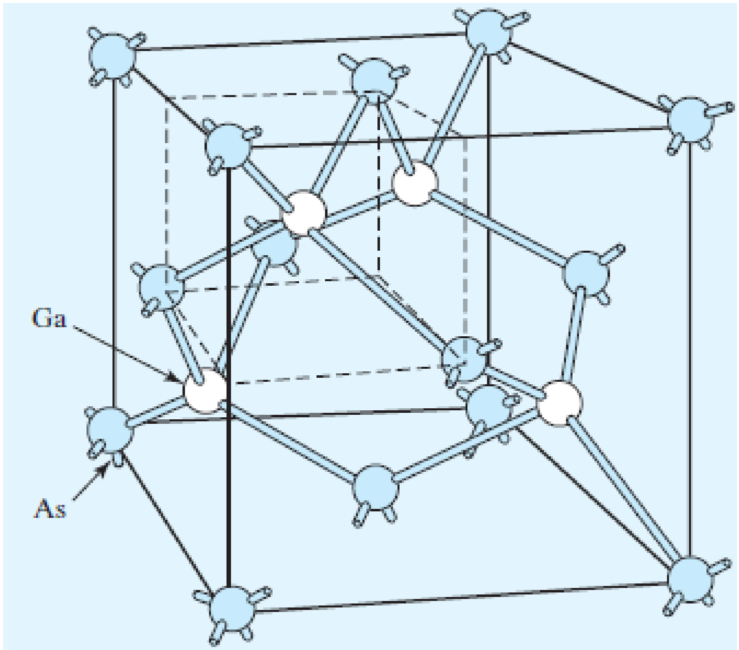
Dopants in Silicon

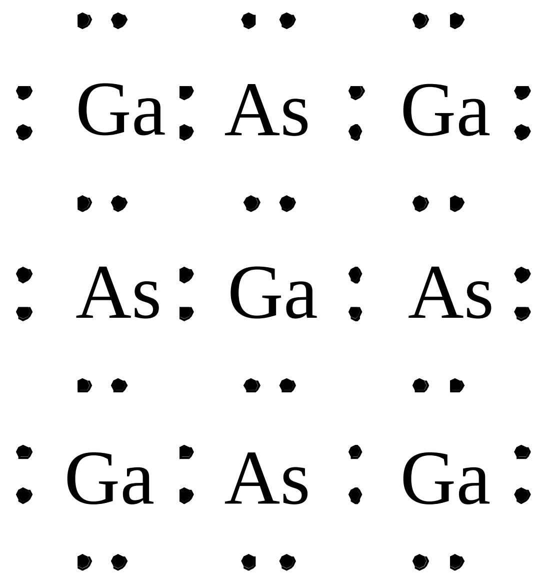


- As, a Group V element, introduces conduction electrons and creates ***N-type silicon,*** and is called a ***donor.***
- B, a Group III element, introduces holes and creates ***P-type silicon***, and is called an ***acceptor***.
- Donors and acceptors are known as dopants. Dopant ionization energy ~50meV (very low).

Hydrogen: $$E_{ion} = \frac{m_0 q^4}{8\varepsilon_0^2 h^2} = 13.6 \text{ eV}$$

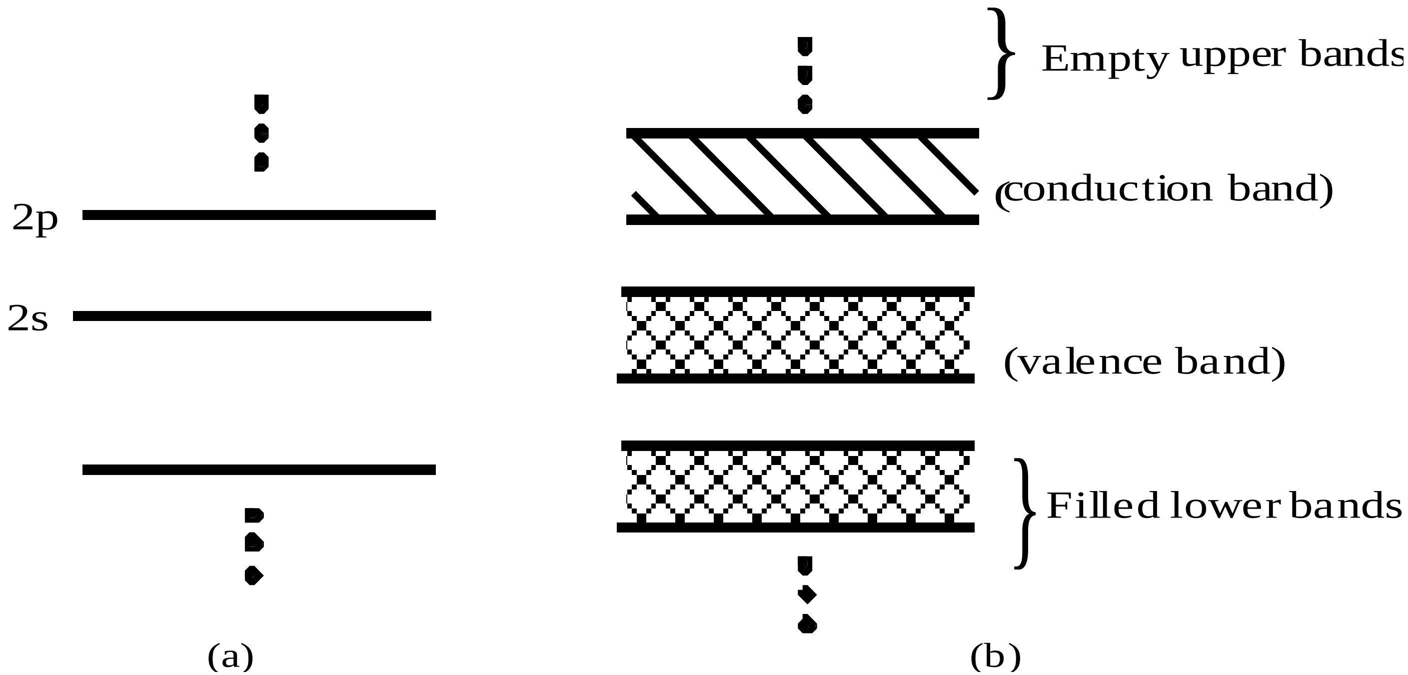
GaAs, III-V Compound Semiconductors, and Their Dopants





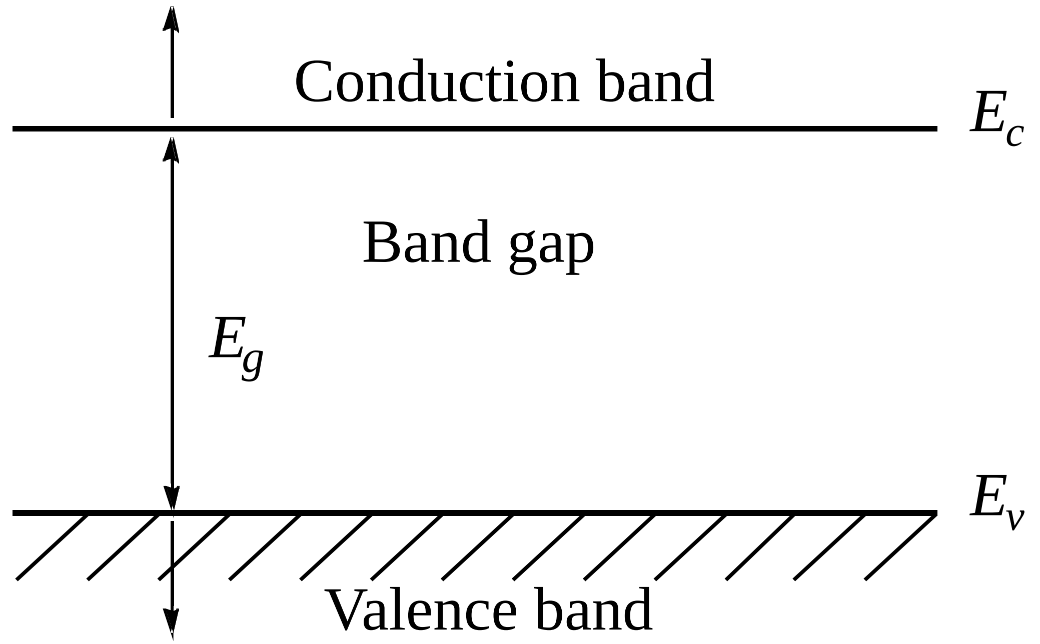
- GaAs has the same crystal structure as Si.
- GaAs, GaP, GaN are III-V compound semiconductors, important for optoelectronics.
- Wich group of elements are candidates for donors? acceptors?

1.3 Energy Band Model



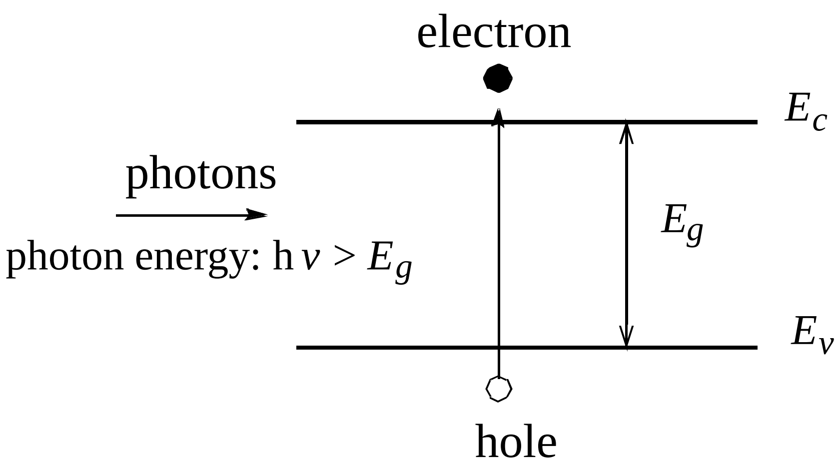
- Energy states of Si atom (a) expand into energy bands of Si crystal (b).
- The lower bands are filled and higher bands are empty in a semiconductor.
- The highest filled band is the ***valence band.***
- The lowest empty band is the ***conduction band*** .

1.3.1 Energy Band Diagram



- ***Energy band diagram*** shows the bottom edge of conduction band, E_c , and top edge of valence band, E_v .
- E_c and E_v are separated by the ***band gap energy, E_g*** .

Measuring the Band Gap Energy by Light Absorption

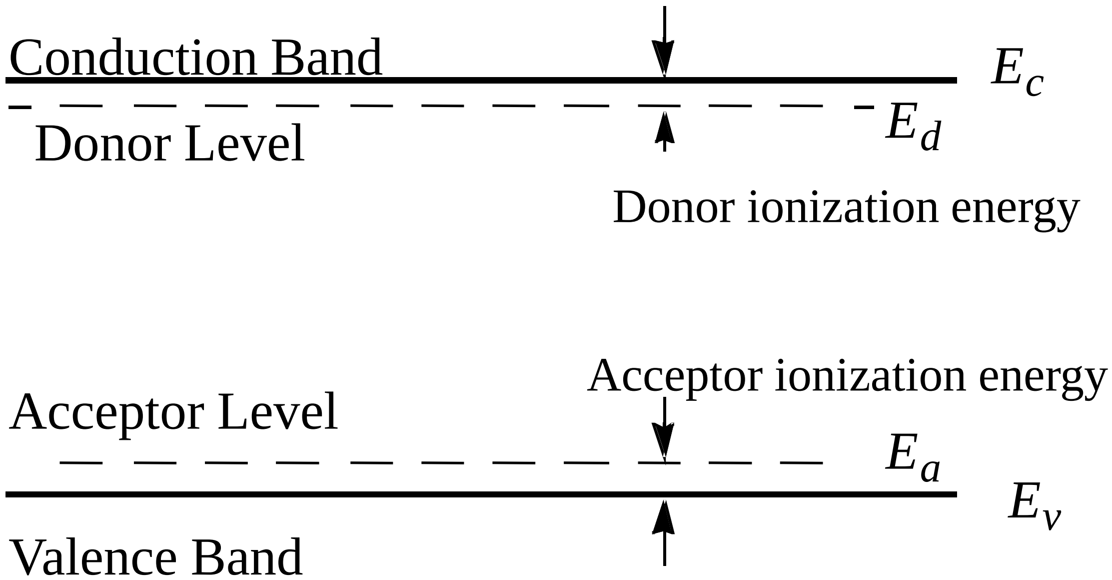


• E_g can be determined from the minimum energy ($h\nu$) of photons that are absorbed by the semiconductor.

Bandgap energies of selected semiconductors

Semi-conductor	InSb	Ge	Si	GaAs	GaP	ZnSe	Diamond
Eg (eV)	0.18	0.67	1.12	1.42	2.25	2.7	6

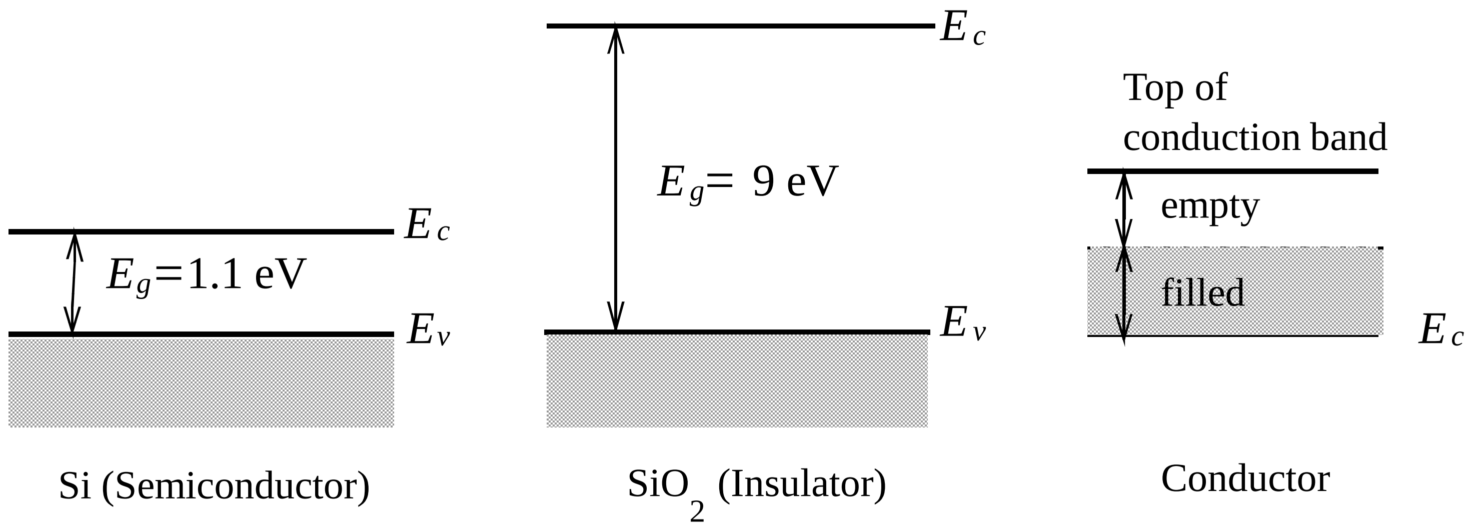
1.3.2 Donor and Acceptor in the Band Model



Ionization energy of selected donors and acceptors in silicon

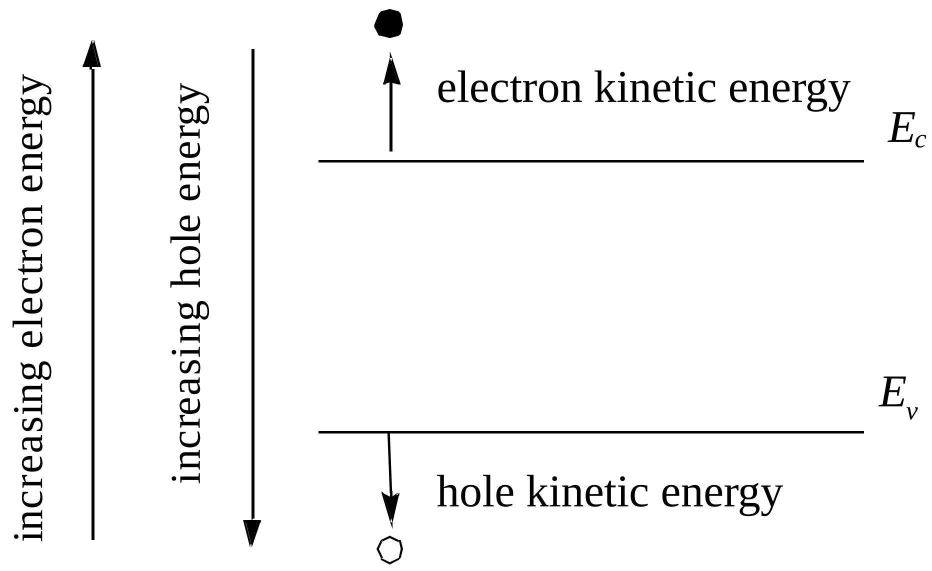
	Donors			Acceptors		
Dopant	Sb	P	As	B	Al	In
Ionization energy, $E_c - E_d$ or $E_a - E_v$ (meV)	39	44	54	45	57	160

1.4 Semiconductors, Insulators, and Conductors



- Totally filled bands and totally empty bands do not allow current flow. (Just as there is no motion of liquid in a totally filled or totally empty bottle.)
- Metal conduction band is half-filled.
- Semiconductors have lower E_g's than insulators and can be doped.

1.5 Electrons and Holes

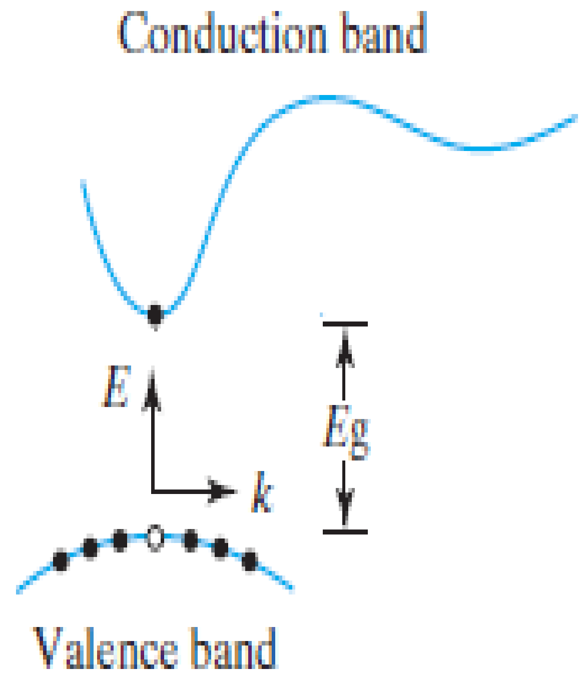


- Both electrons and holes tend to seek their lowest energy positions.
- Electrons tend to fall in the energy band diagram.
- Holes float up like bubbles in water.

1.5.1 Effective Mass

The electron wave function is the solution of the three dimensional Schrodinger wave equation

$$-\frac{\hbar^2}{2\mathrm{m}_0}\nabla^2\psi + V(r)\,\psi = \psi$$



The solution is of the form exp($\pm\,\mathbf{k}\cdot\mathbf{r}$)

k = wave vector = 2π/electron wavelength

For each k, there is a corresponding E.

$$\text{acceleration} = -\frac{q\varepsilon}{\hbar^2}\frac{d^2E}{dk^2} = \frac{F}{m}$$

$$\text{effective mass} \equiv \frac{\hbar^2}{d^2E/dk^2}$$

1.5.1 Effective Mass

In an electric field, **E**, an electron or a hole accelerates.

$$a = \frac{-q\mathcal{E}}{m_n} \qquad \text{electrons}$$

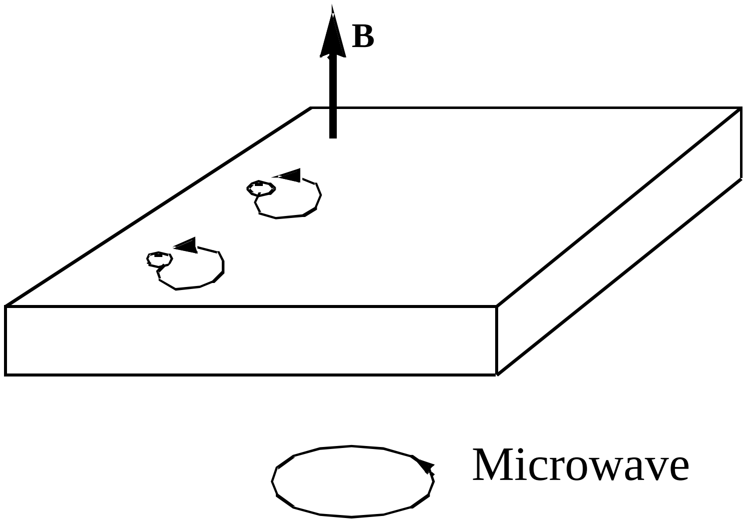
$$a = \frac{q\mathcal{E}}{m_p} \qquad \text{holes}$$

Electron and hole effective masses

	Si	Ge	GaAs	InAs	AlAs
m_n/m_0	0.26	0.12	0.068	0.023	2
m_p/m_0	0.39	0.3	0.5	0.3	0.3

1.5.2 How to Measure the Effective Mass

Cyclotron Resonance Technique



Centripetal force = Lorentzian force

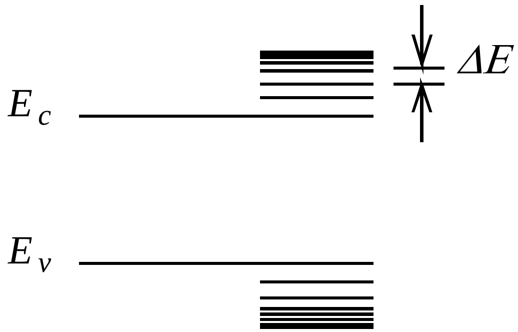
$$\frac{m_n v^2}{r} = qvB$$

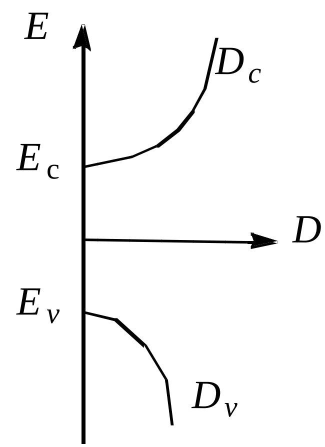
$$v = \frac{qBr}{m_n}$$

$$f_{cr} = \frac{v}{2\pi r} = \frac{qB}{2\pi m_n}$$

- f_{cr} is the Cyclotron resonance frequency.
- It is independent of v and r.
- Electrons strongly absorb microwaves of that frequency.
- By measuring f_{cr}, m_n can be found.

1.6 Density of States





$$D_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \quad \left(\frac{1}{\text{eV} \cdot \text{cm}^3}\right)$$

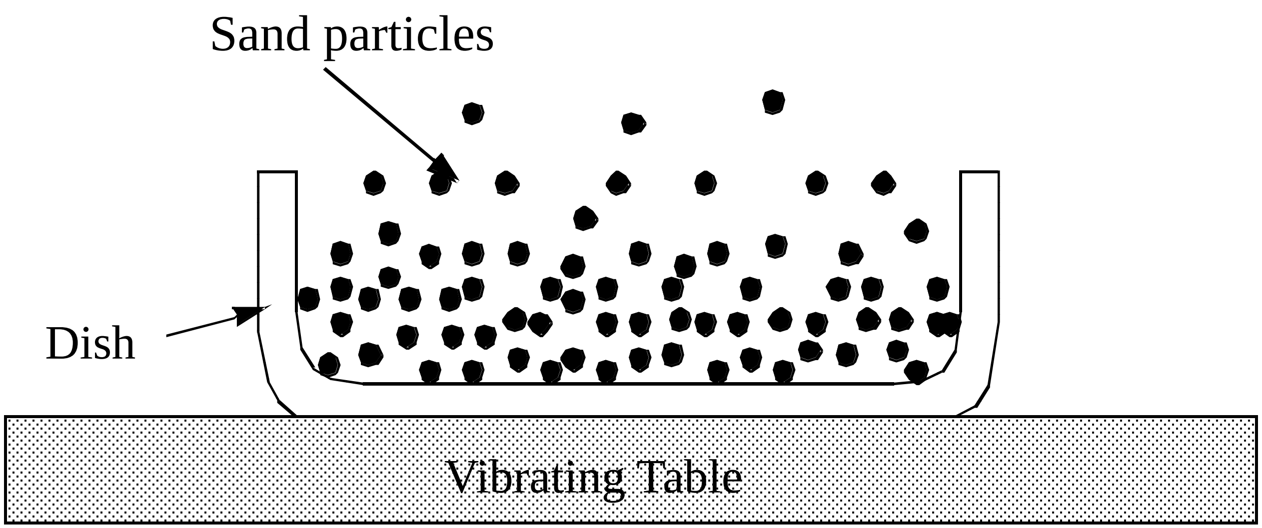
$$D_c(E) \equiv \frac{8\pi m_n \sqrt{2m_n(E - E_c)}}{h^3}$$

$$D_v(E) \equiv \frac{8\pi m_p \sqrt{2m_p(E_v - E)}}{h^3}$$

Derived in Appendix I

1.7 Thermal Equilibrium and the Fermi Function

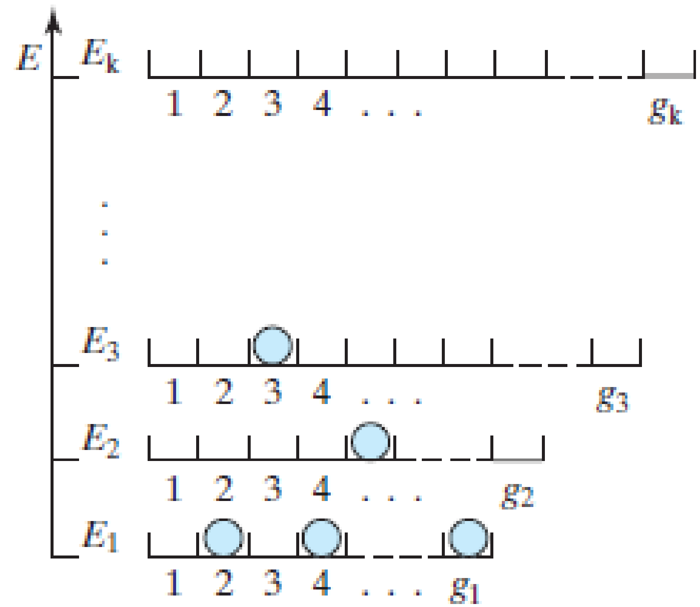
1.7.1 An Analogy for Thermal Equilibrium



- There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy.

Appendix II. Probability of a State at E being Occupied

- There are g_1 states at E_1, g_2 states at E_2… There are N electrons, which constantly shift among all the states but the average electron energy is fixed at 3kT/2.
- There are many ways to distribute N among n_1, n_2, n_3….and satisfy the 3kT/2 condition.



- The equilibrium distribution is the distribution that maximizes the number of combinations of placing n_1 in g_1 slots, n_2 in g_2 slots…. :

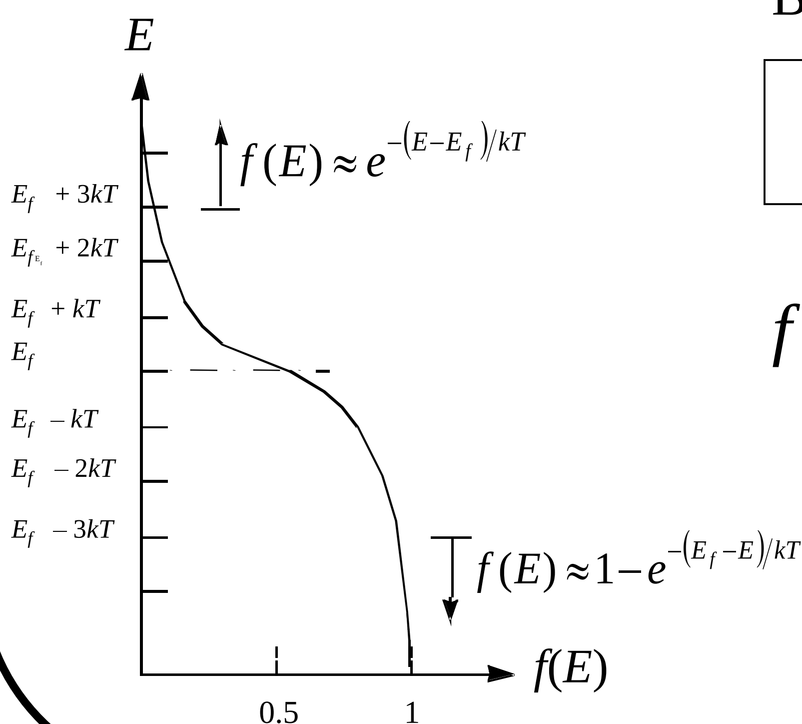
$$n_i/g_i = \frac{1}{1 + e^{(E - E_F)/kT}}$$

E_F is a constant determined by the condition $\sum n_i = N$

1.7.2 Fermi Function–The Probability of an Energy State Being Occupied by an Electron

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$

E_f is called the ***Fermi energy*** or the ***Fermi level.***



Boltzmann approximation:

$$\boxed{f(E) \approx e^{-(E-E_f)/kT}} \qquad E - E_f \gg kT$$

$$f(E) \approx 1 - e^{-(E_f-E)/kT} \qquad E - E_f \ll -kT$$

Remember: there is only one Fermi-level in a system at equilibrium.

1.8 Electron and Hole Concentrations

1.8.1 Derivation of *n* and *p* from *D*(*E*) and *f*(*E*)

$$n = \int_{E_c}^{\text{top of conduction band}} f(E) D_c(E) dE$$

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} \int_{E_c}^{\infty} \sqrt{E - E_c}\, e^{-(E - E_f)/kT} dE$$

$$= \frac{8\pi m_n \sqrt{2m_n}}{h^3} e^{-(E_c - E_f)/kT} \int_{0}^{E-Ec} \sqrt{E - E_c}\, e^{-(E - E_c)/kT} d(E - Ec)$$

Electron and Hole Concentrations

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$N_c \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

N_c is called the ***effective density of states (of the conduction band) .***

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$N_v \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2}$$

N_v is called the ***effective density of states of the valence band.***

Remember: the closer E_f moves up to N_c, the larger *n* is; the closer E_f moves down to E_v , the larger *p* is.

For Si, N_c = 2.8 ´ 10^{19} cm^{-3} and N_v = 1.04 ´ 10^{19} cm^{-3}.

1.8.2 The Fermi Level and Carrier Concentrations

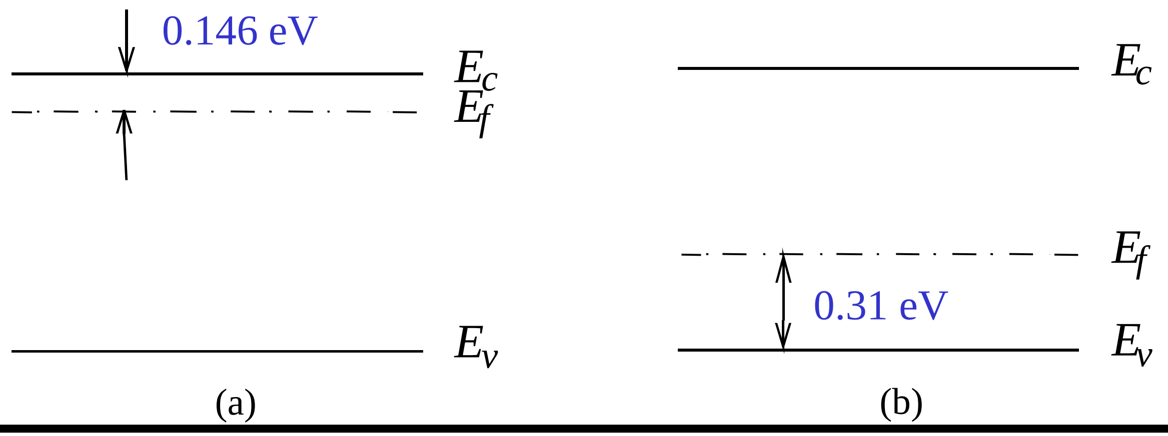
Where is E_f for $n = 10^{17}\ cm^{-3}$? And for $p = 10^{14}\ cm^{-3}$?

***Solution**: (a)* $n = N_c e^{-(E_c - E_f)/kT}$

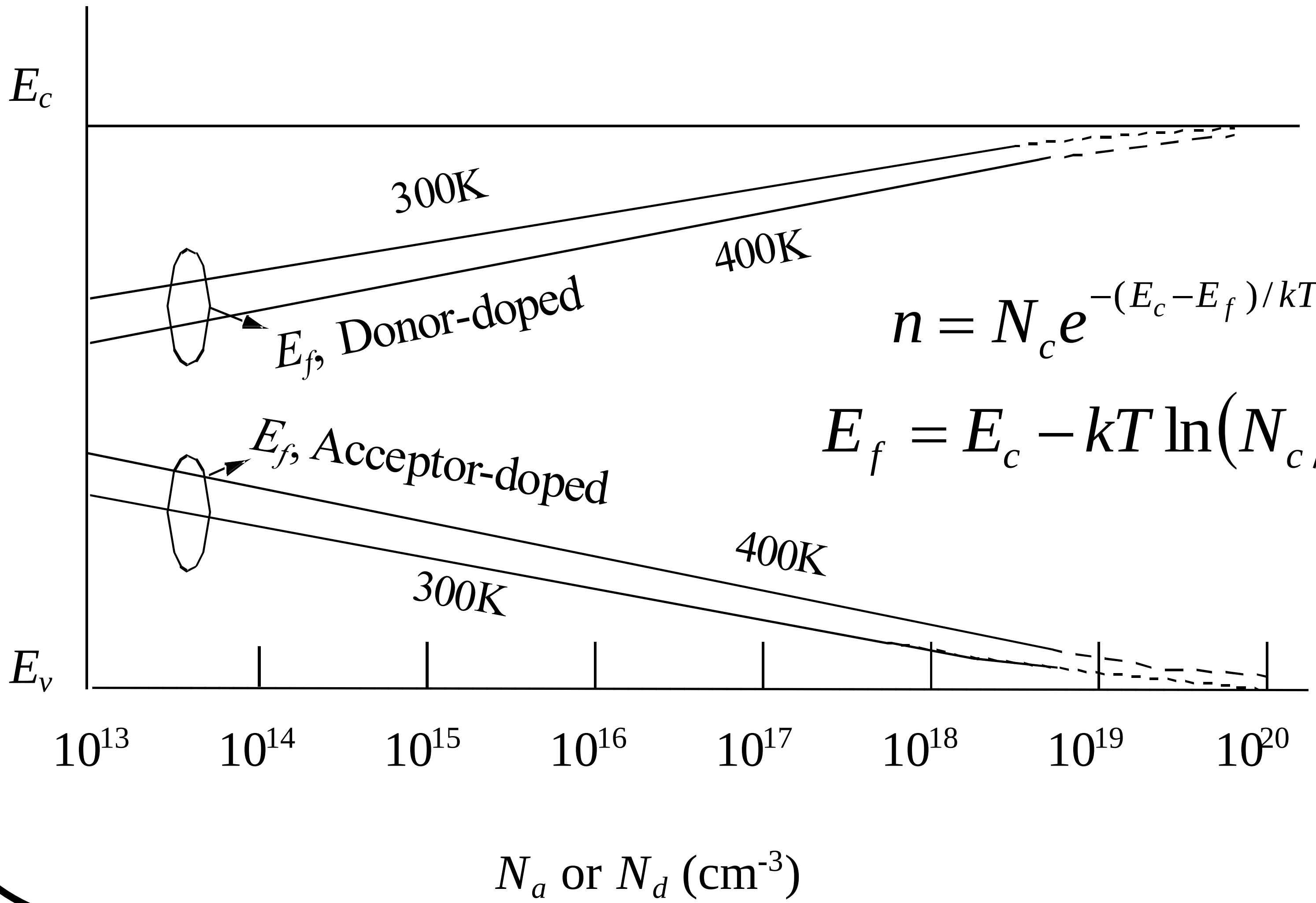
$$E_c - E_f = kT \ln\left(N_c / n\right) = 0.026 \ln\left(2.8 \times 10^{19} / 10^{17}\right) = 0.146\ \text{eV}$$

(b) For $p = 10^{14} cm^{-3}$, from Eq.(1.8.8),

$$E_f - E_v = kT \ln\left(N_v / p\right) = 0.026 \ln\left(1.04 \times 10^{19} / 10^{14}\right) = 0.31\ \text{eV}$$



1.8.2 The Fermi Level and Carrier Concentrations



$$n = N_c e^{-(E_c - E_f)/kT}$$

$$E_f = E_c - kT \ln\left(N_c / n\right)$$

1.8.3 The *np* Product and the Intrinsic Carrier Concentration

Multiply $n = N_c e^{-(E_c - E_f)/kT}$ and $p = N_v e^{-(E_f - E_v)/kT}$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$np = n_i^2$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

- In an intrinsic (undoped) semiconductor, $n = p = n_i$.
- n_i is the ***intrinsic carrier concentration***, ~10^{10} cm^{-3} for Si.

EXAMPLE: Carrier Concentrations

Question: *What is the hole concentration in an N-type semiconductor with 10^{15} cm^{-3} of donors?*

Solution: $n = 10^{15}\ cm^{-3}$.

$$p = \frac{n_i^{\,2}}{n} \approx \frac{10^{20}\,\mathrm{cm}^{-3}}{10^{15}\,\mathrm{cm}^{-3}} = 10^{5}\,\mathrm{cm}^{-3}$$

After increasing T by 60°C, n remains the same at 10^{15} cm^{-3} while p increases by about a factor of 2300 because $n_i^{\,2} \propto e^{-E_g/kT}$.

Question: *What is n if $p = 10^{17} cm^{-3}$ in a P-type silicon wafer?*

Solution:

$$n = \frac{n_i^{\,2}}{p} \approx \frac{10^{20}\,\mathrm{cm}^{-3}}{10^{17}\,\mathrm{cm}^{-3}} = 10^{3}\,\mathrm{cm}^{-3}$$

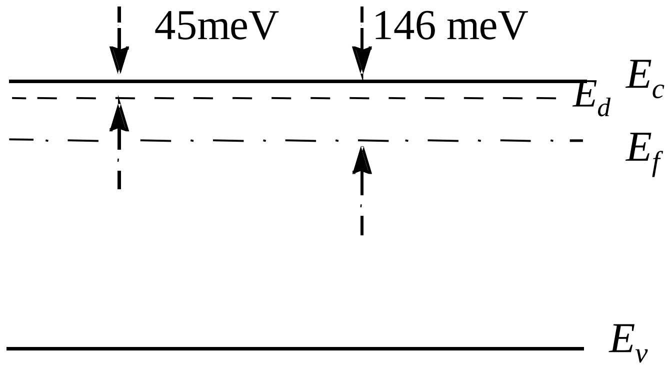
1.9 General Theory of *n* and *p*

EXAMPLE: Complete ionization of the dopant atoms

$N_d = 10^{17}$ cm^{-3}. What fraction of the donors are not ionized?

Solution: *First assume that all the donors* ***are*** *ionized***.**

$$n = N_d = 10^{17}\,\text{cm}^{-3} \Rightarrow E_f = E_c - 146\,\text{meV}$$



Probability of not being ionized $\approx \dfrac{1}{1+\frac{1}{2}e^{(E_d - E_f)/kT}} = \dfrac{1}{1+\frac{1}{2}e^{((146-45)\text{meV})/26\text{meV}}} = 0.04$

Therefore, it is reasonable to assume complete ionization, i.e., $n = N_d$.

1.9 General Theory of n and p

Charge neutrality: $n + N_a = p + N_d$

$$np = n_i^{\ 2}$$

$$p = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2} \right)^2 + n_i^{\ 2} \right]^{1/2}$$

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^{\ 2} \right]^{1/2}$$

1.9 General Theory of on n and p

I. $N_d - N_a >> n_i$ (i.e., N-type)

$$n = N_d - N_a$$
$$p = n_i^{\,2} / n$$

If $N_d >> N_a$, $n = N_d$ and $p = n_i^{\,2} / N_d$

II. $N_a - N_d >> n_i$ (i.e., P-type)

$$p = N_a - N_d$$
$$n = n_i^{\,2} / p$$

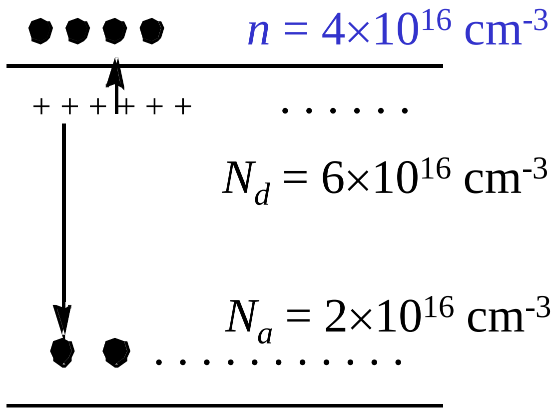
If $N_a >> N_d$, $p = N_a$ and $n = n_i^{\,2} / N_a$

EXAMPLE: Dopant Compensation

What are n and p in Si with (a) $N_d = 6\times10^{16}$ cm^{-3} and $N_a = 2\times10^{16}$ cm^{-3} and (b) additional 6×10^{16} cm^{-3} of N_a?

(a) $$n = N_d - N_a = 4\times10^{16}\,\text{cm}^{-3}$$

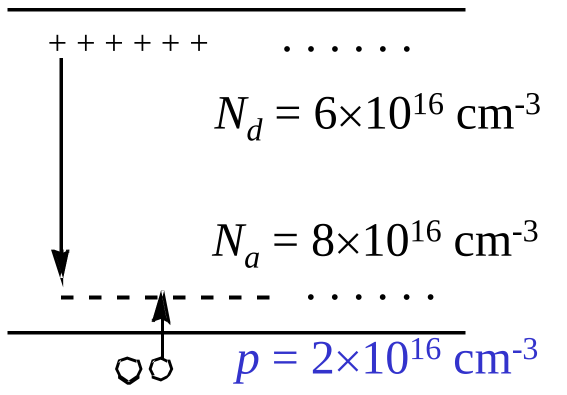
$$p = n_i^2 / n = 10^{20} / 4\times10^{16} = 2.5\times10^{3}\,\text{cm}^{-3}$$



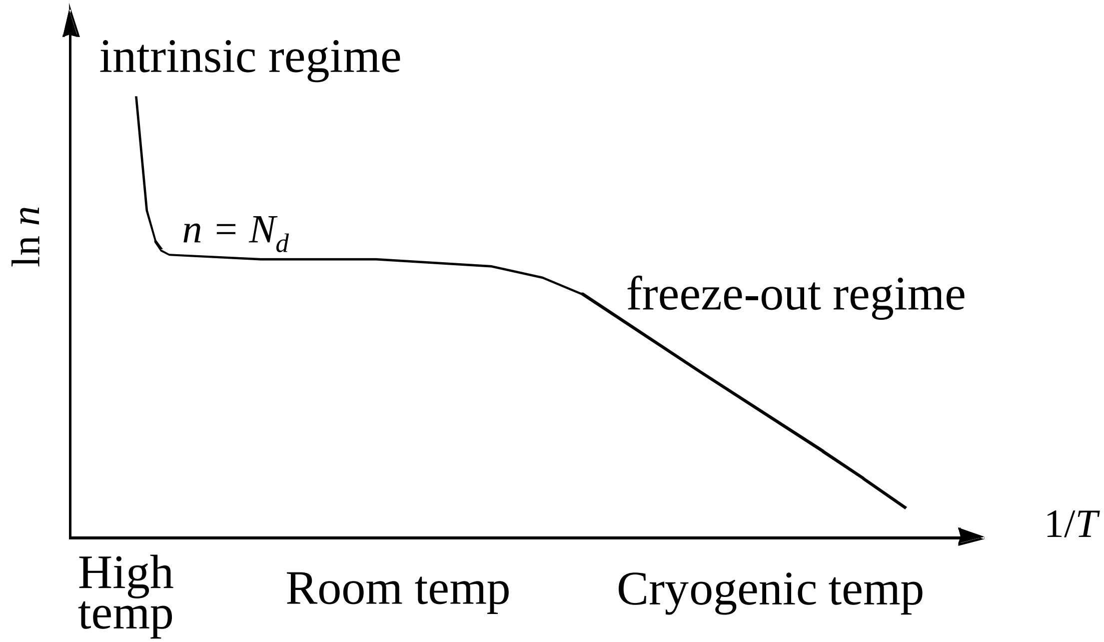
(b) $N_a = 2\times10^{16} + 6\times10^{16} = 8\times10^{16}$ cm^{-3} $> N_d$

$$p = N_a - N_d = 8\times10^{16} - 6\times10^{16} = 2\times10^{16}\,\text{cm}^{-3}$$

$$n = n_i^2 / p = 10^{20} / 2\times10^{16} = 5\times10^{3}\,\text{cm}^{-3}$$



1.10 Carrier Concentrations at Extremely High and Low Temperatures

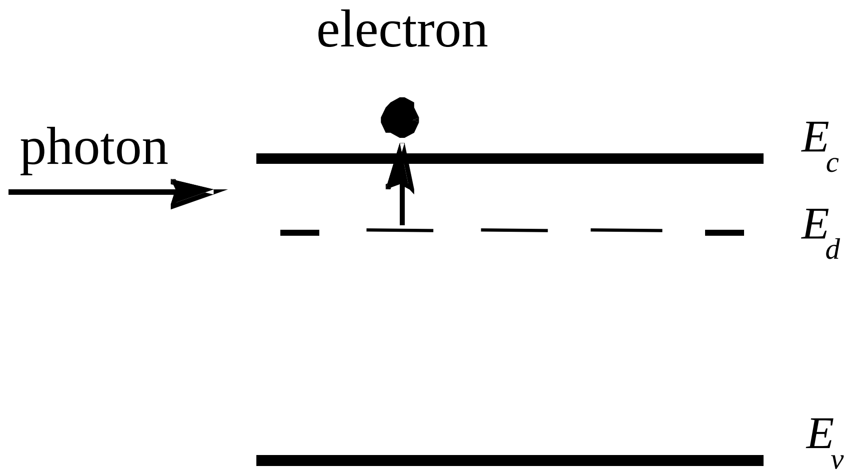


high T: $$n = p = n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

low T: $$n = \left[\frac{N_c N_d}{2}\right]^{1/2} e^{-(E_c - E_d)/2kT}$$

Infrared Detector Based on Freeze-out

- To image the black-body radiation emitted by tumors requires a photodetector that responds to $h\nu$'s around 0.1 eV.
- In doped Si operating in the freeze-out mode, conduction electrons are created when the infrared photons provide the energy to ionized the donor atoms.



1.11 Chapter Summary

Energy band diagram. Acceptor. Donor. m_n, m_p.
Fermi function. E_f .

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$n = N_d - N_a$$

$$p = N_a - N_d$$

$$np = n_i^2$$